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A PROGRESSION FOR GENERATING VARIABLE-INTERVAL SCHEDULES[1]

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In most laboratories, a tape-reading device is used to program variable-interval schedules (VI's) in which the intervals have been derived from the terms in a progression. In the ideal variable-interval schedule, reinforcement (rf) would occur with some predetermined average interval between rf's; but the probability of reinforcement [P(rf)] would not be correlated with any temporal variables.

In the common VI's (those derived from arithmetic and geometric progressions), however, a relationship exists between P(rf) and time since rf. This relationship is illustrated for the hypothetical VI with a mean of 5.5 sec that would be generated from the arithmetic progression of intervals: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 sec. In practice, the sequence of intervals would be randomized on a program tape; and the tape, or tapes, with new randomizations would run for as long as the schedule was in effect. Thus, after an extended period of time, a condition would emerge in which each interval would occur with a fixed relative frequency (1/10). Unfortunately, however, with this progression (as with any arithmetic progression) the P(rf) increases at an increasing rate as a function of time since rf. For example, the P(rf) during the first second since rf is 1/10, since 10 intervals equal or exceed 1 sec; and of these, one is programmed to yield rf at the end of 1 sec. The P(rf) during the 2nd second since rf is 1/9, because an rf occurs during only one of the nine intervals equaling or exceeding the 2nd second. In the same manner, the P(rf) increases with each succeeding second and is unity in the 10th second.

We seek to develop a VI schedule in which rf would occur with a given probability that would remain constant as a function of time since rf. It can be shown that in order for a schedule to exhibit this property, the intervals between successive rf's must be distributed by the following function:

(1) $Pd(t) = -(1-p)^t \log_e(1-p)$ for $0 < t < \infty$

where Pd(t) is the probability distribution of the interval length t, and p is the fixed probability of the reinforcement within a unit interval.[2]

In order to develop a schedule which would satisfy the conditions specified in Equation (1), the number of different intervals must be infinite. Naturally, this condition cannot be achieved within the limitations created by the usual programming equipment, *i.e.*, a limited number of intervals, each occurring with a common relative frequency. However, a progression of intervals can be derived that will provide an unbiased approximation to the ideal conditions. The logic is as follows: consider the set of partitions which will divide the distribution described in Equation (1) into N successive classes of equal area. The relative frequency of the set of intervals within each class is equal to 1/N. If the expected value (mean) of each class is derived and if the mean represents the entire set of intervals within a class, the resulting sequence of means will form a progression with the desired properties: the total number of intervals (means) will equal N; and when these intervals occur with the same relative frequency (1/N), the P(rf) as a function of time since the last rf is roughly constant.

The equation for the progression of means is

(2) $\bar{t}_n = [-\log_e(1-p)]^{-1} [1 + \log_e N + (N-n)\log_e(N-n) - (N-n+1)\log_e(N-n+1)]$

where $\bar{t}_n$ is the n^{th} term of the progression;

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[2]The details of the derivations are given in Research Bulletin (No. 26), issued by the Department of Psychology, The Pennsylvania State University, University Park, Pennsylvania.

N is the total number of terms; and p is the fixed probability of the event within a unit interval.[3]

It is significant that the mean of the theoretical distribution [Equation (1)] is $[-\log_e(1-p)]^{-1}$, and that this term is also the mean of the above progression. Because $[-\log_e(1-p)]^{-1}$ is a constant in Equation (2), it may be replaced by the value of the VI mean (expressed in seconds). To generate a progression of N intervals with a given mean, the *E* would enter the value of N into Equation (2) and replace $[-\log_e(1-p)]^{-1}$ by the value of the VI mean. He would then solve Equation (2) N times, *i.e.*, when $N = 1$, $N = 2 \ldots N = N$. For example, the progression of intervals for VI 30 sec with $N = 12$ is as follows: 1.287, 4.017, 7.020, 10.364, 14.123, 18.423, 23.447, 29.486, 37.067, 47.261, 62.958, and 104.547 sec. The values of the progression with a given N and mean can easily be converted to a progression with the same N and another mean if each term is multiplied by a constant which equals the quotient of the second mean divided by the first.[4]

Consider a VI in which the N different intervals correspond to the terms of the progression. A question arises as to what conditions must exist for such a schedule to have the property that P(rf) as a function of time since rf is constant. It has previously been stated that in order for a schedule to have this property, its intervals must be distributed by the theoretical probability distribution [Equation (1)]. Yet, the probability distribution that emerges for this VI must have a value of zero at all points in time other than the N intervals used. This difficulty would be insurmountable if organisms had perfect temporal discrimination. The fact that they do not means that the effects of rf at a given point in time will spread to nearby points in time (at least within the difference limen). If the differences between successive terms in the progression were sufficiently small so that within the schedule context, discrimination between these terms were poor, the effective probability distribution would be continuous and would approximate the theoretical distribution. Presumably, in any experimental situation, N can be made large enough so that discrimination between successive terms is poor; and, thus, the above property can be obtained by the appropriate use of the progression.

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[3]When $n = N$, the value of $\bar{t}_n$ is $[-\log(1-p)]^{-1}[1 + \log N]$, by L'Hospital's Rule.

[4]A table of values of the progression for selected values of N has been deposited with the American Documentation Institute (Library of Congress, Washington, D.C.), and is also included in the paper described in footnote 2. A copy of this table may be secured by citing Document No. 7108, and by remitting $1.25 for photoprints or $1.25 for 35-mm microfilm. Make checks or money orders payable to: Chief, Photoduplication Service, Library of Congress.